CHAPTER ONE
SOCIOHISTORICAL AND IDEOLOGICAL CONTEXT

The origin of the *Interamerican Committee on Mathematics Education* is closely linked to the reform of Mathematics teaching, a wave that swept most of the world in the 1960s and 1970s. That movement introduced what we now call Modern Mathematics into the realm of general basic education throughout most of our countries[1].

Although some feel that this reform has nothing to do with what is currently going on in Mathematics teaching, it cannot be denied that a sizable number of mathematicians and educators around the world were intellectually nurtured, for better or for worse, within that framework. Moreover, a large number of textbooks and *curricula*, in our schools and colleges, still show very clearly its undeniable influence.

Even though in Europe, in the 1950s[2], there was intellectual concern regarding the teaching of pre-university Mathematics, the initial drive towards reform was given in Edinburgh at the *International Congress of Mathematicians* in 1958[3]. After a report by five American participants[4] representing various groups in the United States, a wave of opinion gave voice to the need for a reform in the methods of teaching Mathematics in Europe[5].

Shortly after, in the fall of 1958, the European Economic Cooperation Administration (EECA)[6] gathered in France a group of representatives from 20 countries. As a consequence of that meeting, the well-known Royaumont Seminar[7] was convened in November 1959. This Seminar established the major guidelines for what became the reform of modern Mathematics. Policies necessary for implementing the reform were also discussed[8].

The contents of the reform are well-known: introduction of set theory; modern symbolism; eradication of Euclidean Geometry; introduction of algebraic structures and axiomatic systems; a functional approach to trigonometry, etc.[9]

The Seminar's war cry was coined by the renowned French mathematician Jean Dieudonné in his opening address: "*Down with Euclid!*"

There were other meetings in subsequent years, with the purpose of carrying ahead the reform: in Arhus, Denmark, 1960 (sponsored by the International Commission on Mathematical Instruction); that same year in Zagrev and Dubrovnik, Yugoslavia; in Bologna, 1962; in Athens (1963) and in Lyon, France (1969), etc.

The reform was always established first in secondary schools, and later in primary schools.

In major European[10] and North American[11] countries, between 1959 and the mid 70s, similar paths were being followed[12]: meetings and conferences; groups of experts in charge of creating syllabi and textbooks, and in-service preparation of teachers. Institutional projects with national or international funding were created for primary schools. After a while, UNESCO[13] began to play an important role. Among the most famous projects were those of Nuffield[14] in England; Alef[15] in Germany, and Analogue[16] in France[17].

The reform appeared in various forms throughout the Third World, and even made headway in the Soviet Union. In just fifteen years New Math had dominated the planet.

If we want to know the reasons for the reform, we must consider several factors and dimensions that may be summarized as follows:

(1) the work of mathematicians from universities,
(2) the ideology and philosophy of Mathematics, and
(3) the political and historical environment of the postwar years.

These three variables intertwined in a very specific way to generate the reform.

The reform was essentially a response to a reality: there was a widely felt need to modernize the teaching of Mathematics, and there was also much concern about the wide gap between university Mathematics and the Mathematics of the secondary school.

Modernization was rooted in the need to adapt mathematical preparation to scientific and technological developments in major Western societies, as well as to some special historical and political conditions.

This situation spurred mathematicians to believe that they had the historical *mission* to involve themselves in pre-university Mathematics education by defining the modernization of school Mathematics and by building an adequate bridge to university Mathematics[18].

The truth is that most national and international conferences were conducted by professional[19] mathematicians[20], many of them internationally recognized in their field[21].

In order to move forward in our considerations we question several assumptions that were accepted at the time, but in retrospective might be challenged:

(i) it is not at all clear if the modernization of Mathematics teaching should be interpreted as the introduction of the contents of modern Mathematics (modernization could have meant an improvement of methods, mechanisms, objectives, etc.)[22]

(ii) it is also not clear if pre-university Math should be defined in terms of the needs of university Mathematics or in terms of the requirements of scientific and technological professions taught in universities.
(iii) it is not clear if mathematicians in the universities (no matter how capable they may be in their field) are really the professionals who should be in charge of defining the mathematics syllabi in the realm of general basic education.

Let us take a look at issues concerning the ideology and philosophy of Mathematics. The immediate theoretical influence controlling the reformers was what could be identified as "the Bourbaki ideology". As is well known, in the 1930's and 1940's, in Nancy, France, a group emerged comprised of outstanding mathematicians. They were guided by the purpose of reconstructing Mathematics on a wide and comprehensive base that would include all the achievements attained, up to that time, in the field of Mathematics.

That impressive organizational task, which generated several dozen Mathematics volumes, was based on the concepts of the set theory, relations, and functions. According to these mathematicians, it was possible to base Mathematics on two mammoth structures: an ***algebraic*** structure and a ***topological*** structure. Each was divided into substructures. The ***algebraic*** structure, for instance, was divided into groups, rings, moduli, fields, etc. The ***topological*** structure was comprised of groups, compact spaces, convex spaces, normal spaces, etc. Both were closely linked to each other through the ***vector space*** structure[23].

This organization of mathematical knowledge became very influential in many universities in various parts of the world, and the same can be said of its assumptions, either explicit or implicit[24].

One of these assumptions is that Mathematics is a unique corpus, and that there exist a language and a conceptual logic that can account for all parts of Mathematics. According to this assumption, the very essence of Mathematics is in its own abstractness and in the creation or expansion of general structures.

The Bourbaki ideology was backed and influenced by many thinkers, even Piaget[25], who found in the structures what he believed was the key to the development of human thinking, not only in terms of *sociogenesis* but also of *psychogenesis*.

This ideology was a decisive driving force for the reformers of pre-university Mathematics teaching.

However, the question we must ask is this: why did this ideology attract so many followers, and so easily, in all parts of the world? What was the force that nourished this ideology? It must be admitted that the members of the Bourbaki group were very prestigious mathematicians, and this carried a lot of weight, but this, by itself, was not enough. It is our contention that the answer is to be found, in a very special way, in the philosophical source that serves as the starting point of this ideology. In other words, the success of the Bourbaki ideology was also a product of the influence exerted upon the

Western mind by philosophical premises about the nature of Mathematics (specifically accepted by the Bourbaki ideology). Let us mention them briefly:

> One constant idea has been to consider that Mathematics is *a priori* knowledge, i.e., not based on experience. Mathematics, therefore, is not an empirical science and, thus, not a natural science (although it can serve science). Mathematical results, therefore, are verifiable through reason, not through experience. This explains why mathematical truths are not merely approximate findings, but absolutes and, therefore, infallible.
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> Another common notion, based on the previous one, holds that abstraction and axiomatization assert themselves as decisive mathematical dimensions, therefore deduction and logical rigor are considered as the essence of mathematical practice.

These ideas were present in the era of the so called "Foundations of Mathematics". They permeated Gottlob Frege's and Bertrand Russell's *Logicism* and David Hilbert's *Formalism*, and some of them even influenced Brouwer's *Intuitionism*.[26]

This poses a problem, since these ideas foster a view that separates Mathematics from sensory experience and other natural sciences, eliminating the role played by empirical intuition and eradicating the heuristic and approximative approach of mathematical practice. Thus Mathematics becomes a pure, abstract, lofty, eternal, absolute and infallible place, to which only the best spirits can ascend.

It should not surprise us, then, that the so-called Platonism in mathematics, which holds that there is a universe of mathematical objects beyond human consciousness, independent of individuals, and perceivable through reason, has found a lot of strength in that vision. (According to that vision, what would be the work of mathematicians? It seems that it is only to describe that abstract universe.) This vision, accepted to a certain degree and adjusted to the particular historical moment, has posed a problem to mathematical practice and is still with us.

These Platonic ideas have prevailed since ancient times throughout the history of Mathematics philosophy. Descartes and especially Leibniz, and also Kant to a certain extent, reconstructed them in modern thought. It can be said that these concepts are part of what is usually called *epistemological rationalism*. Some of them, however, are a part of the traditions of the standard-bearers of neopositivism within the empiricism of the twentieth century.

Let us look at this more carefully. Epistemological Empiricism affirms the preeminent importance of sensory experience in obtaining cognitive truths. In the previous century, Mill used to say that mathematical propositions were inductive generalizations and that the human mind was somewhat like wax on which the reality, from outside of the subject, left its imprint. Mill, of course, was wrong. Mathematical propositions are not inductions nor can the role of the cognitive subject be narrowed that much. We should not accept such an extreme position either. However, positivists in our century, grouped together in the famous Vienna Circle before World War II, exerted a powerful influence on modern philosophy, but could not find a satisfactory answer to Rationalism. Mathematics, for them, has nothing to do with the world and does not refer to the world: it is either pure tautology or a convention of a linguistic or syntactic nature (cf. Carnap's or Ayer's position). These thinkers did not deal effectively with the problem of the nature of Mathematics.

The Bourbaki ideology fit perfectly with the prevailing paradigms[27] concerning the nature of Mathematics, and was, therefore, easy to accept. But, even so, an additional component was still lacking. From an ideological point of view, the reform was very much indebted to Europe[28]. However, in the institutional and financial areas, a great deal was contributed by the United States[29]. One of the factors that influenced the rhythms of reform, and also on the international support it received, bears a Russian name: Sputnik[30]. When the Soviets placed this satellite in orbit, the Western world was frightened, submerged as it was in what is now a thing of the past: the *Cold War*. Sputnik was perceived as evidence of Soviet technological superiority. If things continued like that, soon the Soviets would become masters of the world. The Soviet educational system, which was overrated, was viewed as a dangerous threat to freedom and democracy. History would change that perception, but only a long time after those events.

There was then a general cry to update and improve scientific and technological education in the Western world. The moment was ripe for the reform of Mathematics[31].

Extensive institutional and financial support strengthened mathematics reform[32]. Perhaps the *transparently international* character of the reform was due to that political factor[33].

These concerns about modernization necessarily reached our subcontinent, but the initiative in favor of the reform came from without. First, we received the textbooks of the *School Mathematics Study Group* (SMSG) arriving from the United States. But probably the most decisive event was the *First Interamerican Conference on Mathematics Education*, held in Bogotá, Colombia, in 1961. As we shall see, this Conference received large financial aid from the U.S. National Science Foundation (NSF), and was attended by respected mathematicians, such as Marshall Stone from the United States and Gustave Choquet from France. The participation of representatives from all countries in our hemisphere was sought to implement without delay a strategic plan: preparation and/or translation of textbooks; *curriculum* changes; training of teachers, etc., things that were already going on in Europe and the United States.

A followup conference was held in Lima, in 1966, where the syllabus for secondary schools (12-18 year olds) was prepared. This syllabus would be instrumental in the reform of all Mathematics *curricula* on the subcontinent. Methods and programs for training teachers were also designed in Lima.

Latin America did not have a closely-knit mathematical or scientific community, and this made it easier for the reform to be accepted[34]. Universities got involved in the process, in different ways and at various paces[35], and students returning home after graduating in Mathematics in the United States and Europe, reinforced --in general-- the new plans[36]. Textbooks, sometimes still in use today, played a very important role in that process[37].

The *Interamerican Committee on Mathematics Education* (IACME) was born within this general context[38]. Its first president was the great North American mathematician, Marshall Stone. Luis Santaló, renowned mathematician and educator born in Spain and residing in Argentina, was chosen in 1966 as his representative in everything connected with Latin America. The Committee was in charge of implementing the reform, with representatives from all regions of the hemisphere[39].

As we shall see, the *Interamerican Conferences on Mathematics Education* were faithful for many years to the objectives of the reform. However, just as Mathematics Education kept on changing worldwide, the same thing was happening within IACME. The Reform did much to reinforce the ties among mathematicians all over the world, especially between those in Latin America and their counterparts in the United States, Canada and Europe. IACME became literally an institutional bridge joining the North and the South of the hemisphere in everything dealing with Mathematics and Mathematics teaching. The Reform brought with it a spirit and a mystique among mathematicians, who contributed a great deal in preparing like-minded professionals throughout the entire region, and strengthening their academic realm within the universities.

Whether or not they were based on ideas that are correct, many of the actions that were born in the initiatives that sprang up around the reform have contributed to developing the professionalization of the Mathematics teachers as specialists, in their own right, different from the mathematician and the general educator.

With the passage of time, the Reform's original objectives disappeared from IACME, just as had happened elsewhere. However, an international organizational framework has remained. It must be recognized as the most permanent and important one in the field of Mathematics Education in Latin American in the last thirty years.

Notes

1 Some fragments of this chapter appeared for the first time in the article written by Angel Ruiz, "Las matemáticas modernas en las Américas: Filosofía de una Reforma" [Modern Mathematics in the Americas: Philosophy of a Reform], *Educación Matemática*, vol 4, no. 1, April 1992. It was also published by UNESCO in the book *Las Matemáticas en las Américas VIII*, Paris, 1993.

2 Surveys in this respect had been carried out by UNESCO and OECD before Royaumont; this can be seen in UNESCO reports from 1950 and 1956.

3 You can consult the work by Howard Fehr, John Camp and Howard Kellogg: *La revolución en las matemáticas escolares (segunda fase)* [The Revolution in School Mathematics], Washington DC: OAS, 1971, p. 8.

4 It is interested to point out just who were those representatives: Marshall Stone, Albert W. Tucker, E. G. Begle, Robert E. K. Rourke, and Howard F. Fehr. Stone, Fehr, and Begle were later to be involved with IACME.

5 *Ibid*., p. 9.

6 This organization was based in Paris. Today it is called the Organization for Cooperation and Economic Development (OCED).

7 From November 23 to December 4 of 1959, in the Cercle Culturel of Royaumont, Asnieres-sur-Oise.

8 *Ibid*, p. 9.

9 The Royaumont Seminar culminated a process of 4 or 5 years of interest in the modernization of pre-university mathematics.

10 In France, cradle of the Bourbaki group, the reform agenda was developed as follows: 1955: preparatory classes for the "Grandes Ecoles"; 1963: reform of the last years of secondary; 1969: all of secondary; 1971: the first years of primary school. See *L'ecole en proie a la mathématique*, cahiers pédagogiques 110, Janvier, 1973, p. 7.

11 In the USA during the 50s there were many reform initiatives in the school mathematics programs. Of course, in 1958, before Royaumont, the National Science Foundation supported a conference for mathematicians in Chicago; and a week later there was a similar meeting in Cambridge, Massachusetts. See Moon, Bob: *The "New Maths" curriculum controversy. An international story*, London: The Falmer Press, 1986; p. 46.

[12] When recommended by circumstances institutional educational systems were used.

[13] The role of UNESCO can be clearly seen in mathematics education by the second half of the 60s. The creation of the Center for Educational Research and Innovation (CERI), in 1968, revealed such a direction; a study of the reports associated with that center provide a mechanism for examining the course of reform: it can be said that the crucial years of UNESCO support were 1969 to 1974.

[14] The director of Nuffield was Geoffrey Matthews.

[15] In 1965 Heinrich Bauersfeld was designated to direct the project on school mathematics; and in 1966 the project *Alef* was launched at the University of Frankfurt in Hessen.

[16] Project *Analogue* was directed by Nicole Picard.

[17] With respect to primary education various conferences can be mentioned: in Stanford, USA, in December of 1964; in Paris in April of 1965; and in Hamburg in January of 1966. All were organized by the *International Group for Mathematics*, created in 1962, and supported and financed by UNESCO. In those years one of the individuals that most helped to publicize the reform was Z.P. Dienes. Cfr. Moon, op. cit., p. 55.

[18] As Moon pointed out: "The case studies demonstrate that one interest group appears to have been particularly influential in the early years of reform. The impact of university mathematicians, notably those advocating a "bourbakist" reform of the school *curriculum*, is demonstrated in each country", *op. cit.*, p. 216. Moon is referring to France, Holland, England, Germany, and Denmark.

[19] Some with certain links to government. See Moon, *op. cit., p. 198.*

[20] Some of the most important reformers in Europe were Bauersfeld in Germany, Christiansen in Denmark, Freudenthal in Holland, Picard in France, and Matthews in England. Only Matthews did not come from a university.

[21] In the USA there was a concerted effort directed by mathematicians: E.G. Begle was named to direct the School Mathematics Study Group (SMSG) supported by the American Mathematical Society, the Mathematics Association of America, and the National Council of Teachers of Mathematics; cfr. Moon, *op. cit.*, p. 46.

[22] It is interesting to point out that one of the critics of the reform (although he was a *suis generis* reformer in Holland) was Hans Freudenthal. In fact, from the 50s he had come out against the introduction of modern mathematics; he was the one who had spoken instead of a modern teaching of mathematics. One of his last critical articles was "New Maths or New Education", *Prospects*, vol. 9, no. 3, p. 321-331, 1979.

[23] See Fehr, *et. al.*, p. 29.

[24] Without a doubt the Bourbaki ideology had an influence in the USA; see Moon , *op. cit.*, p. 65.

[25] Piaget even collaborated with an important publication that reunited the principal reformers in France: see Lichnerowicz, A., Piaget, J., Gattegno, C., Dieudonné, J., Choquet, G., Beth, E.W., in *L'Enseignement des mathematiques*, Delachaux and Niestle, 1960.

[26] For an extensive study of these themes, see the book by Angel Ruiz: *Matemáticas y Filosofía. Estudios Logicistas.* San José, Costa Rica: Editorial de la Universidad de Costa Rica, 1990.

[27] Without a doubt, a process was at work in modern mathematics reform similar to those that in science Kuhn has called paradigm shifts. A paradigm was created that was supported by a very large and heterogeneous community, in which mathematics played a central role. The new paradigm deteriorated in a few years, without having yet created a substitute paradigm.

[28] More than imposition from Europe or imposition from the United States, the matter should be seen as a parallel process of innovation where reciprocal influences were present. Cfr. MacDonald, B. and Walker, R., *Changing the Curriculum*, London: Heinemann, 1976.

[29] It should be remembered that it was in the USA that the strategy of R & D was developed that tried, among other things, to bring about changes in *curricula*.

[30] That satellite was launched on October 4, 1957.

[31] We should be careful here, no matter how important Sputnik was historically, it was not the only factor, or the determining factor, in the reform. See Moon, *op. cit.*, p. 65.

[32] As soon as Sputnik was launched the Madison Project was created in the USA; in Canada the Sherbrooke Mathematics Project was created; in England an educational commission under the direction of Sir Geoffrey Crowther was created; 7 years later the Nuffield Mathematics Project was created for the primary level. Cfr. Moon, *op. cit.*, p. 146.

[33] That can be seen in the composition of the Royaumont Seminar. Perhaps it should be recalled that international collaboration was strengthened precisely in those years: in 1960 Canada and the USA entered the OCEE forming the OECD.

[34] The case of Costa Rica is interesting because the reform was codified in official programs beginning in 1964; this was due to a special situation: the Costa Rican educational system experienced a reform in the early 60s; Dr. Alfaro Sagot, took advantage of the circumstances to introduce the main aspects of reform into the mathematics program of 1964. Alfaro himself wrote the first textbooks with the new focus, although it should be pointed out that completely abandoning intuitive aspects and a relation to physics.

[35] The process of preparing mathematics teachers in Latin America essentially was developed in the 70s; and it was dominated by Bourbakian paradigms and rationalist philosophies. It is necessary to take into account this situation in the moment of delineating plans for the future.

[36] Many of them also helped to create a distancing between mathematics and mathematics education, as well as, between mathematics and the other sciences.

[37] Of course, on the international level a mathematics textbook industry was generated, provoking an extraordinary socialization to the new mathematics.

[38] See *Educación Matemática en the Américas. Informe de la Primera Conferencia Interamericana sobre la Educación de las Matemáticas* [Mathematics Education in the Americas. Report on the First Interamerican Conference on Mathematics Education]; edited by Howard Fehr, Teachers College, Columbia University: Bureau of Publications, 1962, p. 184.

[39] The reform influx into Latin America benefitted from a particular experience in Chile, Argentina and Uruguay that can be symbolized by the creation of the Consejo Latinoamericano de Matemáticas e Informática (CLAMI) [Latin American Council of Mathematics and Informatics]. The special relationship of Argentine intellectuals to Europe promoted in particular the special intervention of the Bourbaki group in Latin America: Dieudonné himself taught a course for several months in Buenos Aires to young mathematicians that came from various parts of South America and that, later, would be influential professionals in Latin American mathematics. IACME was not the only route travelled by Bourbaki ideology on its way to Latin America.