Mathematics and Academic Diversity in Japan

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Abstract

Japanese education has been the subject of considerable research and educational commentary in the United States over the last 20 years. Since the early 1990s there has been increased interest in Japanese methods for teaching mathematics, and the Third International Mathematics and Science Study has accelerated American interest in Japanese methods. Observational studies, teacher and student surveys, and analyses of classroom videotapes have provided a rich picture of how the Japanese teach the whole class. However, little has been written about how academically low achieving math students fare in Japanese schools. This article briefly summarizes Japanese methods for teaching mathematics and then describes how the educational system addresses academic diversity. It concludes with a description of a method for teaching mathematics that some Japanese mathematics educators feel has promise for students with learning disabilities.
American assessments of Japanese education since the 1960s have been of two minds. Educators, journalists, and politicians in the US have produced a mix of opinions that vary widely and, at times, are wholly contradictory. Singleton’s (1967) ethnography of Japanese secondary schools, for example, presents a picture of drill-oriented teaching and passive students. More recent accounts (e.g., Yoneyama, 1999) offer similar, if not darker descriptions of the secondary education experience. Bracey (1996) and Wolferen (1989) argue that this dark side of the education system is hidden from (or overlooked by) American researchers who are highly supportive of Japanese instructional methods. Researchers (Berliner & Biddle, 1995, Westbury, 1992) even question the superior performance of Japanese students on international comparative exams by suggesting that only the best students in the country are allowed to take these tests. They also argue that high test scores in subject areas like mathematics reveal little about broader abilities in problem solving and creativity that might be stifled by a rigid, test-oriented educational system.

Contrary accounts of Japanese education also have appeared over the same period of time. For example, returning from a fact finding mission sponsored by the Reagan administration, prominent American educators offered enthusiastic accounts of Japanese education and how it should be adopted in the United States (see Berliner & Biddle, 1995). A decade later, Stevenson and Stigler’s (1992) popular press book, *The Learning Gap*, attempted to dispel many of the negative stereotypes of Japanese education cited above. The authors used mathematics not only as an index for comparing academic achievement, but also as a way to present Japanese teaching practices in a constructive light. The authors also drew attention Japan’s reluctance to identify students as learning disabled and in need of special education services. Citing the high monetary and emotional costs of special education, Stevenson and
Stigler argued that Japan offers a clearer vision of what it wants from their educational system, and the results have been evident in higher academic outcomes for all students in that country.

Research in the 1990s, particularly the range of work conducted as part of the Third International Mathematics and Science Study (TIMSS), has led to a steady shift toward even more positive views of Japanese education. To some extent, these views have emerged because American researchers have had a stronger cultural and linguistic understanding of Japan (White, 1999). The competitive economic advantages that can be accrued from high levels of scientific and mathematical literacy may be another reason for a closer, more detailed examination of Japanese educational practices (LeTendre & Baker, 1999).

This article draws on a range of research conducted by American and Japanese researchers. At many points, we make direct comparisons to American education as a way of elucidating relatively unique features of the Japanese system. The article begins with an overview of mathematics education in Japan. The second section of the article describes how the Japanese educational system addresses academic diversity as well as the extent to which learning disabilities is a viable construct. The final section will describe the Suido Method of instruction. This method is used on some occasions to teach mathematics to struggling students and those suspected of having learning disabilities.

Learning Mathematics in Japan

The 1995 TIMSS studies have been instrumental in addressing a criticism commonly lodged at Japanese education. That is, that superior performance of Japanese and other Asian students on earlier international mathematics tests was a result of sample bias (i.e., only the top students in these countries participated in comparative studies). TIMSS was designed to address this problem, and the majority of TIMSS participants cooperated in recruiting representative
students from their country’s population. Test score rankings in mathematics at the fourth and eighth grade tests indicated that Japanese students were significantly higher than US students at both grade levels. On the eighth grade test, for example, Japan was ranked third on the list of 40 countries. The US was 28th, and it was only significantly higher in its performance than seven participating countries (Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999).

TIMSS research attempted to account for differences between countries in more ways than statistical analyses of test scores. Analyses of curricula, teacher and student surveys, and videotapes of classroom practices have enabled researchers to investigate factors which may have contributed to differential performance on the TIMSS test (see Schmidt, McKnight, & Raizen, 1997; Schmidt et al., 1996; Stigler & Hiebert, 1999).

Curricular materials. Schmidt et al. (1997) characterized American mathematics curricula as reflecting “many goals.” A range of agencies -- reform organizations like the National Council of Teachers of Mathematics, state curriculum adoption panels, local school boards – all shape mathematics instruction at the classroom level. One consequence of the interaction of different forces is that curricular materials tend to be, “a mile wide and an inch deep.” Typical American texts present a great number of topics, but only superficially. Unlike other, more successful countries that participated in the TIMSS, American curricula repeat the same topics across more grade levels with only modest increases in the difficulty of the concepts. The texts can also contain a significant array of optional activities, making them the physically largest texts found in the TIMSS study (Schmidt et al., 1997; Schmidt et al., 1999).

In contrast, Japanese math texts, which follow strict guidelines established by the Japanese Ministry of Education or Monbusho, contain far fewer topics. Over the years, texts follow a “concentric ring” pattern, with each new grade level providing greater depth on a topic.
Over grades one through eight, Japanese curricula move students into relatively complex mathematics, while US curricula largely emphasize arithmetic. Figure 1 contrasts the number of mathematics topics as they develop over time in Japan and the US. Notice the divergence in topics once students in both countries begin grade 6, which is the end of elementary school in Japan and often the start of middle school in the US.

A detailed analysis of TIMSS items corroborates this structure to Japanese curriculum. When items are separated by content, Japanese students score significantly better than US students on many, but not all topics. This is particularly true at the fourth grade level, where US and Japanese students are comparable in certain aspects of geometry (e.g., position and shape; symmetry, congruence, and similarity). Variations in performance across topics suggests that Japanese math curricula put greater emphasis on areas such as two dimensional geometry; proportionality; measurement; patterns, relations, and functions; and data representation and analysis (see Schmidt et al., 1996; Schmidt et al., 1999).

In comparison to American texts, Japanese math curricula appear to be spartan. The texts are slim volumes, often printed in paperback. There are few pictures, little in the way of supplemental or optional materials, and by comparison to US curricula, relatively few practice problems. By implication, it is assumed that well-established pedagogical practices will compensate for what is not presented in the textbook (Stevenson & Stigler, 1992).

Finally, the organization of mathematics classes at the secondary level also contributes to the wide array of topics that are presented in textbooks. TIMSS data indicate that almost half of the American students were in schools that offered at least three or more different math classes at the eighth grade level. By comparison, 99 percent of Japanese eighth graders were in schools
that only offered one mathematics course. This difference attests to the role of tracking in US middle schools and the consequential effect that American eighth graders are taught a range of mathematics based on ability.

Pedagogical practices. Schmidt et al. (1999) observed that TIMSS teachers throughout the world tend to implement a common set of global teaching practices. Math teachers review material from previous lessons, go over homework, provide instruction on new concepts, have students work on exercises, and allow time for homework. One of the major differences between Japan and the US is how time is distributed across these practices. American teachers, particularly secondary educators, spend more time going over homework and allow more time for homework to be done during class than their counterparts in Japan. American teachers also spend less time going over new concepts and allocate far less time for students to work on classroom exercises or thought-provoking problems.

Some of the best descriptions of how Japanese teachers conduct lessons can be found in a number of studies of elementary school mathematics. Lee, Graham, and Stevenson (1998), for example, describe carefully implemented practices passed along from experienced to novice teachers through extensive apprenticeships once new teachers are hired in schools. Lessons begin slowly and build methodically, with an attempt to engage students in a challenging problem. A much greater emphasis is placed on thinking about the problem than quickly coming up with a solution.

Students are then typically directed to work on one or a few problems in small groups. At the elementary level, students use Sansu Setto (mathematical kits that contain tools and manipulatives) as part of the group work. Manipulatives and visual aids are considered important elements in representing and eventually, discussing mathematical solutions. Students
work together for varying periods of time, but TIMSS research indicates that Japanese students are four times more likely that American students to work on exercises or challenging problems for 15 minutes or more (Schmidt et al., 1999).

Japanese teachers eventually interrupt the small group work to discuss different solutions to the problem(s). They go to considerable lengths to extend a student or group’s point of view, regardless of its correctness. This alone is a striking difference between Japanese and US classrooms. US teachers are exceedingly reluctant to present errors or “incorrect thinking” in classes as the basis for discussion, a phenomenon some argue is largely a function of the behaviorist tradition in American schooling (Stevenson & Stigler, 1992; Tsuchida & Lewis, 1998).

Japanese teachers will move back and forth from the whole class to group work, using whole group discussions as a way of raising questions and discussing inconsistencies in student thinking. Lee et al.’s (1998) comparative study of a Sendai, Japan, and Chicago classrooms indicates that Japanese teachers tended to engage students in alternative perspectives on a problem three times more often than the American teachers.

Furthermore, the way classroom discussions punctuate the student exercises enables teachers to give feedback on “embedded seatwork.” Japanese teachers use these discussions as an opportunity to elaborate on the day’s topic. In contrast, US teachers tend to follow the prescription of allowing students to work on seatwork or homework for an extended period of time at the end of the lesson. This idea, which was a key recommendation from process product research in the 1980s (Brophy & Good, 1986; Good, Grouws, & Ebmeier, 1983), subtracts a significant amount of interactive instructional time from the lesson.
A number of research reports provide similar accounts of Japanese math lessons, particularly the ones conducted at the elementary school level (e.g., Stevenson, 1991; Stevenson & Stigler, 1992; Stigler & Hiebert, 1999; Stigler, Fernandez, & Yoshida, 1998). What is striking about Japan’s comparative success in mathematics, at least in respect to this article, is that it occurs with class sizes that are much larger than those in the United States and with little or no formal programs for remedial students or ones with learning disabilities.

Addressing Academic Diversity

How Japanese students can attain such high levels of success in comparison to US students has been enigmatic to many American researchers. Attempts to explain differences between American and Japanese children based on intellectual abilities (e.g., Lynn, 1982) have been soundly criticized. Stevenson and his colleagues (e.g., Stevenson, Stilger, Lee, Lucker, Kitamura, & Hsu, 1985; Uttal, Lummis, & Stevenson, 1988) conducted a series of studies that indicated no major differences in cognitive abilities between American and Japanese students and that differences in the area of mathematics that favored the Japanese were best attributed to the home and school.

One might further posit that Japan’s relatively homogeneous culture is somehow reflected in its academic performance. Hence, the success of Japanese students on international comparison tests may be related to relatively low variability in student performance on the tests. As Figures 2 and 3 indicate, data from the TIMSS show a different story. The boxplots indicate a wide range of performance in both the US and Japan at the fourth and eighth grade levels on TIMSS.

<Insert Figures 2 and 3 about here>
While understanding how Japanese educators address academic diversity is a complex affair, one thing is fairly clear. The country’s current educational system does not provide services for academically low achieving students or students with learning disabilities in any way comparable to what is done in the United States. Goldberg’s (1989) discussion of special education in Tokyo indicated that only 0.6 percent of students received services in either special schools or special education classes in regular schools. As one might expect, these were students with low incidence disabilities. Okano and Tsuchiya (1999) estimate that 0.5 percent of the pupil population in the entire country was in special education. Finally, Tsuge (2001) suggested that slightly over 1 percent of the school age population is in special education.

What follows is a brief description of how academic diversity is addressed in Japanese schools. Japanese schools follow a 6-3-3 model, which was established after World War II based on American policies. At each level of school, different factors exist that affect the performance and academic path of struggling students. This section concludes with a brief summary of the current state of learning disabilities in Japan.

**Elementary school.** It is argued that the Japanese educational system addresses some aspects of diversity before children enter elementary school. Japan has one of the most highly developed preschool systems in the world, and approximately 70 percent of children attend either public, private, or national nursery schools (Boocock, 1991; Lewis, 1989). The nursery school years for three to five year olds is seen as a critical transition period, where young children are gradually socialized into important school related behavior (Peak, 1991). It is in preschool that students practice routines that will be used throughout elementary school. Teachers delegate authority to children, and young students learn to work with others and identify with the group or *han.*
Strong working relationships with the *han* become important in elementary school because everyone is expected to participate in classroom activities and discussions. This is viewed as a responsibility, not an option (Lewis, 1988; Sato, 1993; Tsuchida & Lewis, 1998). Teachers also configure groups so that they are composed of diverse abilities. Lee et al. (1998) found that the teachers in their study saw great benefit to mixing slow learners with faster learners so that both types of students could benefit. Using ideas that are sometimes expressed in constructivist literature, teachers felt that low achieving students were in a position to watch and listen to more capable peers. The latter children benefited because they needed to explain their mathematical thinking to the slow learners. Okano and Tsuchiya (1999) also attest to this strategy of mixed ability grouping among elementary age students.

As students move across elementary grades, two powerful traits are developed which affect academic performance. *Gambaru* (roughly translated as “effort”) and *gaman* (persistence) are characteristics of Japanese students that stand out to Western observers. Singleton (1989) notes that these traits reflect more than the specific effects of the educational system. They pervade the culture, and parents are important in instilling them in their children. *Gambaru* is seen as important to success in school and other areas of life.

Blinco’s (1991) comparative research helps validate these constructs as factors that may contribute to increased academic success across a range of students. In a study of Japanese and American children involving a puzzle-like game with a series of increasingly difficult stages, Japanese children persisted significantly longer than the American children. The type of school (private, public urban, public rural) and the interaction between type of school and culture were non-significant in this study.
Stevenson and Stigler (1992) see the emphasis on effort that permeates Japanese education as one of the key reasons why academically low achieving students do better in school than their American counterparts. Rather than slowing down the curriculum or directing struggling students to alternative programs of instruction (e.g., Title I services, special education, low track classrooms), students are encouraged to put in extra effort to keep up. Finally, Lee et al.’s (1998) study of elementary school mathematics found that Japanese teachers were three times more likely to work with students who needed additional help outside of class than the American teachers in their comparison group.

**Middle school.** The middle school years mark a clear shift in the Japanese student’s educational experience. Instruction intensifies and there are an increasing number of tests that assess an individual’s mastery of content. Even middle school teachers complain about the pace of the curriculum (Trelfa, 1998). While there is little evidence that a significant number of elementary students participate in after school study classes (Russell, 1998), there are indications that enrollment in study schools or *juku* increased markedly in the middle school years (Harnisch, 1994). There is much confusion around the nature of these after school institutions, and Westerners tend to associate *juku* with the intense preparation for high school and university, what is often called “examination hell” (Amano, 1989; Frost, 1991).

In fact, *jukus* serve many different functions, from enrichment activities to examination preparation. *Gakushu juku* provides remedial assistance by helping struggling students keep pace with instruction at the middle school level. These *jukus* specialize in mathematics, English, Japanese, and science instruction, and they provide the kind of individualized instruction not found in the school (Fukuzawa, 1998; Rohlen, 1980; Trelfa, 1998). Even though *gakushu juku* may not be a solution for students with the greatest academic needs, they help students –
particularly urban students – to keep up with the intensity of middle school subjects like mathematics. Recent data suggest that these remedial practices are commensurate with worldwide trends associated with modern schooling (Baker, Akiba, LeTendre, & Wiseman, 2001).

**High school.** The last three years of school in Japan are not mandatory. However, 97 percent of students continue on to high school. Retention rates are equally high, ranging from 91 to 99 percent (see Okano & Tsuchiya, 1999; Tsuge, 2001). It is the high school entrance exams administered in eighth grade that give Japan the reputation as a stressful, test-driven instructional system. For example, these exams can determine entrance into elite high schools that in turn, prepare students for admission to top universities. However, there is no single eighth grade exam, and entrance requirements vary among the different types of high schools. It is conceivable that students who are doing poorly in school, ones who will not attend the highly academic or elite track high schools, do not experience the kind of pressure associated with eighth grade exams. Moreover, some Japanese researchers question the accuracy of this stereotype, suggesting that the drive for high levels of performance in education is part of a more complex cultural phenomenon (e.g., Stevenson, 1989; Stevenson & Stigler, 1992).

Okano and Tsuchiya’s (1999) analysis of high schools in 1995 indicates that students who struggle in school are likely to attend three prominent types of schools: 1) evening high schools (2.2 percent of the student population), 2) correspondence high schools (1.6 percent), or 3) schools for students with severe auditory, cognitive, physical, or emotional disabilities (0.2 percent). Okano and Tsuchiya found that evening high schools were for students who work during the day or students who had high absentee rates in school. These schools tended to enroll students with low academic abilities. Correspondence high schools were designed for flexibility,
and they served students who had high absentee rates or problems adjusting to regular high schools. It is logical, then, that Japan accommodates academically low achieving students through these kinds of high schools.

**Students with learning disabilities.** These characterizations of elementary, middle, and high schools do little to elucidate what, if anything is done to meet the needs of students with disabilities. After all, it is reasonable to expect that there are students in Japan who fit the classical description of a student with learning disabilities in mathematics as they are portrayed in the US (e.g., Geary, 1994).

The problem of finding and describing services for students with learning disabilities may be due to the fact that learning disabilities is only a recent phenomenon in Japan. For example, just over a decade ago, Goldberg (1989) cited the concerns of Tokyo educators at the lack of sophisticated assessments that could be used to identify and serve students with learning problems. Tsuge’s (2001) recent discussion of learning disabilities in Japan indicates that trial efforts to serve students with learning disabilities only began in 1990. He further states that learning disabilities is problematic because it conflicts with the traditional special education framework, which tends to serve only students with the most severe disabilities. According to Tsuge, attempts to serve students with learning disabilities will require a more coordinated or “blended” effort with regular education under a new, inclusionary model.

More telling in Tsuge’s (2001) analysis is that over the last decade, learning disabilities has been an issue restricted largely to policy discussions. The Monbusho and the National Institute of Special Education have simply spent much of the last ten years studying the problem and issuing reports. One report by the National Institute of Special Education (National Institute of Special Education, 1995 cited in Tsuge, 2001) indicated that 10 percent of fifth and sixth
grade students were two or more years behind in mathematics. The report also contained a definition of learning disabilities that resembled the US definition of learning disabilities in mathematics -- a difficulty in acquiring a specific ability like calculating or reasoning with no basic delay in general intellectual development. According to Tsuge, widespread interventions for learning disabilities in areas like mathematics are still forthcoming.

A driving force for services for students with learning disabilities at the school level appeared in a recent Monbusho report on gakkyu hokai or “classroom collapse.” Not all Japanese classrooms function in the highly ordered, compliant fashion found in stereotypes or in the han (cooperative groups) described earlier in this article. Instead, teachers can lose control over their classrooms and are unable to conduct lessons effectively. The recent report indicated that students with learning disabilities and ADHD had contributed to the gakkyu hokai phenomenon. All of this has renewed appreciation for individual differences as a top priority for current educational reform in Japan.

The Suido Method

The Monbusho and the National Research Institute on Education in Japan are still at the initial phases of developing generalizable interventions for students with learning disabilities. Nonetheless, one technique -- the Suido Method (Ginbayashi, 1984) for teaching mathematics -- is now recognized among practitioners as an effective method for the children with learning disabilities. The Suido Method was conceived in 1958, and was part of a broader movement by the Association of Mathematical Instruction to develop more effective methods for teaching mathematics in the post World War II period in Japan.

The Suido Method addresses three main concerns: 1) the relationship between mental and written calculations, 2) relations between the concept of number and the four basic operations,
and 3) what principles should guide the calculation system. Regarding the first question, children frequently learn to count numbers, but this activity does not necessarily help them learn quantity or “magnitude.” The Suido Method uses tiles to make quantities explicit to students. Ten small tiles can be combined into a bar to represent 10, and ten of these bars comprise one large square sheet that shows 100. These representations can be used to show decimal numbers. By working with tiles, children come to associate quantity with positional notation (e.g., four 10 bars is 40) and the written symbols. Furthermore, students should generally learn written calculations before they do the rounded, mental calculations that we use in everyday life.

The second principle follows from the first. Once students have been shown the interrelationship between the tiles, place value, and the written symbol, they can be shown each operation on the number. At the time that the Suido Method was developed, textbooks contained many ways of teaching the basic operations, including instruction in the four basic operations simultaneously in a controlled domain of numbers (e.g., up to twenty) (Ginbayashi, 1984). The Suido Method underscores the importance of teaching the four operations separately as different calculations and that tiles can be an effective mechanism for showing basic concepts such as uniting, taking away and distributing.

The third concern of the Suido Method had to do with guiding principles of instruction. Two principles are central: generality--speciality and analysis--synthesis. Generality has to do with what is not only the most common, but what may be easier for students to understand. For example, suppose students had to compute the answer to the following two problems:

\[
\begin{array}{c}
54 \\
+ 42 \\
\end{array} \quad \begin{array}{c}
30 \\
+ 20 \\
\end{array}
\]

Mathematicians who developed the Suido Method argue that the first problem is easier for students to understand through written calculation, and that the second problem (30 + 20) is a
special case because it involves zeros. A written calculation’s complexity is in its form, not in the size of the calculation numbers (Ginbayashi, 1984). For Westerners, this concern may not be obvious, but as Figure 4 demonstrates, representing the two problems through tiles shows how the lack of tiles in the one’s place can possibly confuse the struggling student. As a consequence, problems with zeros much be treated as special cases.

<Insert Figure 4 about here>

The second principle of analysis and synthesis involves carefully decomposing a complex process into its elementary steps, teaching those elementary steps, and then combining them in what the Suido Method calls a “compound process.” To some, this may sound like task analysis as it is used in special education in the United States. A problem such as the one below entails specific, elementary components.

<Insert Figure 5 about here>

The elementary steps of adding each of these single digit numbers involve basic calculations. However, they need to be combined with a strong sense of place value, and that makes it a compound process. At this point, the principle departs from what might typically be derived from task analysis. Figure 5 shows how tiles would be used to demonstrate how numbers could be decomposed to “make 10” first. In this way, the students see the place value in the calculation and the need for regrouping.

<Insert Figure 6 about here>

This process, then, is an analysis of a task into its elementary parts and then a subsequent synthesis of these parts. Suido Method instruction continues beyond basic operations to fractions, decimals, and other secondary topics. To insure mastery, teachers are encouraged to
provide students with carefully controlled practice on problems as they move from general to special cases.

The developers of the Suido Method, the Association of Mathematical Instruction, are also concerned about the number of students who grow to dislike mathematics and drop out of math classes. To that end, they have created a series of “enjoyable lessons” that develop math concepts in game like contexts. Ginbayashi (1984) describes a lesson for teaching the coordinate system using a 6 x 6 chessboard and chips or milk bottle caps. Figure 6 shows one possible configuration of chips on the board.

A student roles a die twice, and he or she moves from the lower left corner across (i.e., the X axis) using the first number rolled and up (i.e., the Y axis) using the second number. If there is a chip on the square, he or she collects the chip and roll again. If not, the other player takes his or her turn.

The Suido Method borrows its name from the “Water Supply Method.” This metaphor refers to the way that water flows from a main source through branches from village or house to the next. By identifying key elementary processes, ordering them, and then combining them in compound processes, mathematical problems “flow” into more coherent forms for students.

It should be noted that the Suido Method contains many of the characteristics found in contemporary Japanese education. There is a significant emphasis on visual representations and the careful analysis of mathematical problems. Unlike the high emphasis on procedural fluency in arithmetic found in mathematics curricula for students with learning disabilities in the US (see Woodward, Baxter, & Scheel, 1997), there is significant attention to important concepts such as place value.
The extent to which an approach like the Suido Method will become institutionalized for teaching students with learning disabilities in Japanese schools over the coming decade is uncertain at this point at time. As mentioned earlier, the Monbusho and the National Research Institute on Education are still at the beginning phase of developing generalizable interventions for students with learning disabilities. These efforts come at a time when the entire educational system from primary grades through higher education is in flux. The half-day of school on Saturday has just been eliminated from the school calendar, and the Monbusho has proposed a number of Western-style changes for the coming decade, ones that will promote greater deregulation and decentralization of schools. Policy changes include ability grouping, promoting the inclusion of students with disabilities into regular schools, and easier methods for suspending delinquent students (Hayo, 2002). These are radical proposals for a system that, by international standards at least, is considered to be a model of effective education. The extent and nature of instructional services for students with learning disabilities that will emerge by the year 2010 is uncertain at this point in time.
References


Figure 1

Number of Math Topics in Japan and the United States*

* Adapted from Schmidt et al., (1997)
Figure 2

Boxplots for Japan and the United States on the Fourth Grade TIMSS*
Figure 3
Boxplots for Japan and the United States on the Eighth Grade TIMSS*

* Adapted from Schmidt et al., (1999)
Figure 4
Addition Using Tiles
Figure 5

Suido Addition Problem

\[
\begin{array}{c}
457 \\
+ 326 \\
\end{array}
\]

\[
\begin{array}{c}
4 \\
5 \\
7 \\
+ 3 \\
+ 2 \\
+ 6 \\
\end{array}
\]
Figure 6

“Making 10” from 7 + 6

(7 ones) → (5 + 2)

(5 + 2) + (5 + 1)

(10 + 3)
Figure 7
Learning the Coordinate System through a Game